THEORETICAL NOTE

Why φ, 5, and 17 Appear Together: A Possible Connection to SO(10) Grand Unification

Speculative hypothesis — no derivation from a Lagrangian is provided

Judicael Brindel, Independent Researcher, Cotonou, Benin
ORCID: 0009-0007-4590-9874

February 2026

Abstract. In previous work [1,2], we observed that five electroweak and flavour mixing observables are expressed within experimental uncertainties using only the integers 5 (bosons in the Standard Model) and 17 (total elementary particles), and the golden ratio φ. This note proposes a speculative but coherent explanation: the integers 5 and 17 are natural numbers of the SO(10) grand unification group, and φ emerges from the rank-5 pentagon geometry of SO(10). If correct, the pattern 5–17–φ is not a numerical coincidence but a structural signature of SO(10) symmetry breaking. No Lagrangian derivation is provided.

1. The Observation to be Explained

The following five observables have been found to lie within their experimental uncertainties when expressed using only 5, 17, and φ:

- $\sin^2\theta_W = 5/(17\sqrt{\varphi}) \rightarrow 0.033\sigma$
- θ_{12} leptons: $\tan(\theta) = 5\sqrt{5}/17 \rightarrow 0.105\sigma$
- θ_{23} leptons: $\cos(\theta) = 5\sqrt{5}/17 \rightarrow 0.171\sigma$
- θ_{13} leptons: $\theta = 4\pi/(5\times17) \rightarrow 0.534\sigma$
- θ_{12} quarks (Cabibbo): $\tan(\theta) = 5/(17\sqrt{\varphi}) \rightarrow 0.416\sigma$

The question is: *why do the integers 5 and 17, and the golden ratio φ, appear together in these five independent observables?*

2. SO(10) as a Natural Source of 5, 17, and φ

SO(10) is the leading candidate group for grand unification. It has three properties directly relevant to our observation:

2.1 Rank 5 → the integer 5 → φ

SO(10) has rank 5. The rank of a Lie group determines the number of simultaneously conserved quantum numbers (Cartan subalgebra dimension). In the Standard Model after symmetry breaking, 5 bosons survive: the photon, W^+, W^-, Z, and the Higgs.

Crucially, rank 5 connects to the pentagon: a regular pentagon has 5 sides, and its diagonal-to-side ratio is exactly $\varphi = (1+\sqrt{5})/2$. More precisely:

$$\mathbf{\cos(36°) = \varphi/2, \quad \cos(72°) = 1/(2\varphi)}$$

These are exact algebraic identities. The golden ratio is the eigenvalue of the pentagon's geometry. Therefore:

$$\mathbf{\text{rank}(SO(10)) = 5 \Longrightarrow \text{pentagon} \Longrightarrow \varphi}$$

This is the proposed reason why φ appears in the mixing angles: it is the geometric signature of the rank-5 structure of SO(10).

2.2 The Spinor 16 + Higgs = 17

SO(10) contains a fundamental spinor representation of dimension 16. This accommodates exactly 16 fermions per generation: the 15 Standard Model fermions plus one right-handed neutrino. Including the Higgs boson:

16 (spinor SO(10)) + 1 (Higgs) = 17

This is precisely the denominator in our formulas. The integer 17 is the natural fermion content of one SO(10) generation plus the Higgs.

2.3 The Ratio 5/17

Combining the two observations:

- 5 = rank of SO(10) = surviving bosons after symmetry breaking
- 17 = spinor dimension + Higgs = matter content per generation
- 5/17 = boson fraction of the SO(10) spectrum

The ratio 5/17, combined with the pentagon geometry φ, produces the mixing angles. The formula $\sin^2\theta w = 5/(17\sqrt{\varphi})$ would then read: the weak mixing angle is the boson fraction of SO(10), corrected by the pentagon geometry of the rank-5 Cartan subalgebra.

3. Internal Consistency

Several features of this hypothesis are internally consistent:

- **Pentagon and heptadecagon:** Ciborowski (2025) independently arrives at 5 and 17 via constructible polygons. The pentagon (5 sides, related to φ) and the heptadecagon (17 sides) are the only Fermat prime polygons needed. This is consistent with SO(10) rank-5 → pentagon → φ.
- **Koide formula:** The Koide constant K = 2/3 for lepton masses corresponds to $\tan(\theta) = 1/\sqrt{2}$, where θ is the angle between a cube diagonal and its faces. The cube has 8 vertices = 2^3. In SO(10), the spinor 16 = 2^4. The cube is the rank-3 analog of what the 16-dimensional spinor is at rank 5.
- **The product 5×17 = 85:** appears in $\theta_{13} = 4\pi/85$, encoding both the rank (5) and the spinor+Higgs (17) directly as a product.
- **Planck mass:** m_Planck/me = (1/α)^(10+13/17φ), where 10 is the dimension of SO(10) spacetime embedding (string theory) or the rank of SO(10)×2, and 13/17 = (spinor-bosons)/spinor+Higgs.

4. What Is Missing

This note is purely speculative. The following steps are required to make it rigorous:

- A derivation of $\sin^2\theta w = 5/(17\sqrt{\varphi})$ from the SO(10) Lagrangian or its symmetry breaking chain
- An explanation of why √φ (not φ itself) appears in the Weinberg angle
- A derivation of the PMNS and CKM angles from SO(10) representation theory
- Identification of the geometric object whose angle is 4π/85
- A connection between the Koide cube and the SO(10) spinor

Without these derivations, the hypothesis remains an observation — suggestive but not proven.

5. Summary

We propose that the pattern 5–17–φ observed in five mixing observables has a structural origin in SO(10) grand unification:

$$\mathbf{5 = rank(SO(10)) \rightarrow pentagon\ geometry \rightarrow \varphi}$$

$$\mathbf{17 = dim(spinor_{16}) + 1 = matter\ content\ per\ generation}$$

$$\mathbf{5/17 \times f(\varphi) = mixing\ angles}$$

This is a hypothesis, not a derivation. It is presented here to establish priority and to invite theoretical investigation by the community.

References

[1] J. Brindel, "A Numerical Observation on the Weak Mixing Angle $\sin^2\theta w = 5/(17\sqrt{\varphi})$", Zenodo (2026).

[2] J. Brindel, "The 5–17–φ Pattern in Electroweak and Flavour Mixing Angles", Zenodo (2026).

[3] J. Ciborowski, "Bi-Constructible pattern of weak and flavour mixing", arXiv:2508.00030 (2025).

[4] Y. Koide, Lettere al Nuovo Cimento 34, 201 (1982).

[5] PDG 2024, Particle Data Group. https://pdg.lbl.gov

Working note, February 2026. Priority established by date. Internet Archive deposit pending.